Social Cooperation and Disharmony in Communities Mediated through Common Pool Resource Exploitation

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(Received 6 July 2016; revised manuscript received 16 March 2017; published 16 May 2017)

It was theorized that when a society exploits a shared resource, the system can undergo extreme phase transition from full cooperation in abiding by a social agreement, to full defection from it. This was shown to happen in an integrated society with complex social relationships. However, real-world agents tend to segregate into communities whose interactions contain features of the associated community structure. We found that such social segregation softens the abrupt extreme transition through the emergence of multiple intermediate phases composed of communities of cooperators and defectors. Phase transitions thus now occur through these intermediate phases which avert the instantaneous collapse of social cooperation within a society. While this is beneficial to society, it nonetheless costs society in two ways. First, the return to full cooperation from full defection at the phase transition is no longer immediate. Community linkages have rendered greater societal inertia such that the switch back is now typically stepwise rather than a single change. Second, there is a drastic increase in social disharmony within the society due to the greater tension in the relationship between segregated communities of defectors and cooperators. Intriguingly, these results on multiple phases with its associated phenomenon of social disharmony are found to characterize the level of cooperation within a society of Balinese farmers who exploit water for rice production.

DOI: 10.1103/PhysRevLett.118.208301

Phase transition is an important topic in physics. While its theoretical foundation and experimental verification are well developed in major branches of physics such as condensed matter physics and statistical mechanics [1,2], it is currently under active investigation within complex systems [3,4]. Nonetheless, there are good theoretical and empirical correspondences in the latter development. For example, phase transition between the observed free-flowing phase to the jammed phase in traffic systems has been nicely explained by fundamental diagrams determined theoretically [5,6]. Empirical epidemic spreading of diseases (or information) from the susceptible phase to the infected phase is well described by complex network models with diverse contagion mechanisms [7,8]. Even fireflies adopt a phase of synchrony or incoherent flashing according to a process analogous to the Kuramoto model [9]. Recently, phase transition has also been uncovered in ecological systems [10–12], banking systems [13,14], and the state of our mental health [15].

In this Letter, we investigate the phase transition of a coupled social-resource system consisting of interacting agents that exploit a common pool resource (CPR). The agents are subjected to social norms as they engage in economic activities employing the shared resource to optimize their individual utility. Depending on the level of resource inputs, the system either resides in the cooperative phase where agents abide by the social rule with resource depletion, or the defective phase where agents disobey the rule, but with resource abundance. The study of phase transition between these extreme phases had been performed in a society that is socially integrated [16–18]. While these investigations have thrown light on the emergence of cooperative actions in coupled social-resource systems, they have yet to relate empirically to a real-world system. The qualitative understanding gained is nonetheless important as it demonstrates how a CPR can be sustained without the need of centralized governance—a phenomenon in line with Ostrom's theory on a CPR [19]. It is also relevant to environmental sustenance issues such as widespread deforestation, species extinction, and pollution at the planetary scale [20–22], whereupon the CPR system has been shifted to a phase of resource depletion.

In this Letter, we have gone one step further by bridging the gap between theoretical and empirical correspondences in a coupled social-resource system by examining the social structure of communities in the form of social segregation, which is a natural fabric of society according to Schelling's theory of segregation [23,24]. While individuals within a community form close ties with each other, different communities may tend to avoid social contact in order to protect their identities and cultures [25]. In network theory, such a tendency is described by the network's community structure [26]. Here, we have uncovered new intermediate phases that mitigate the drastic critical transition from one extreme phase to another in lieu of the limited cross-community interactions. The elucidation of the consequential multiple phase transitions has enabled us to understand how the decline and revival of social cooperation in one community affect the sustainable and cooperative behavior of another community in terms of inter- and intracommunity relationships. In addition, as the state of the system shifts between multiple phases, one observes the manner in which social disharmony progresses at both the community and societal level. These results are consistent with empirical data from a real-world system, the Balinese rice terraces, and explain the multiple phases displayed in this coupled social-resource system.

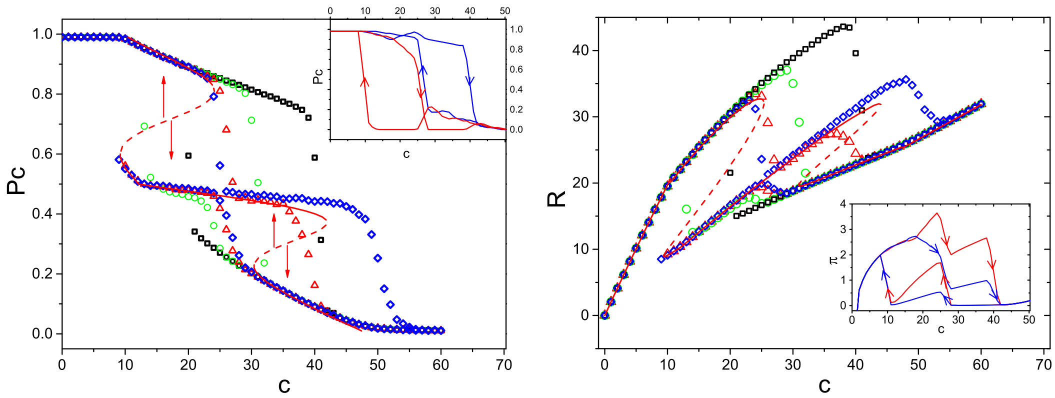
To begin, let us consider a society where every individual relies on a common pool resource. In our model, each individual is represented by a node in a complex network with community structure and homogeneous degree distribution of mean degree $\langle k \rangle$ [26], with social interaction between agents corresponding to a link in the network. The network consists of two community groups of equal size that are tightly connected internally, while the groups can have a different degree of interconnection between them depending on their level of interrelatedness. This relationship between groups is expressed in terms of a mixing parameter μ [27], which quantifies the probability that the link of a node is connected to its external communities.

Next, we adopt the model of Tavoni, Schlüter, and Levin [28]. The resource R in the model is assumed to have a finite capacity $R_{\max}$. It is supplied by a constant inflow c and depreciated by an amount d via natural processes. The time evolution of the resource dynamics is given by $\Delta R/\Delta t = c - d(R/R_{\max})^2 - ER$, where $E = P_c e_c + (1 - P_c) e_d$ is the mean effort. In this model, we consider two forms of social strategy: being a cooperator or a defector, with P_c the probability of cooperators and $P_d = 1 - P_c$ the probability of defectors in the society. A cooperator extracts the resource with effort e_c by abiding to the social agreement, while a defector puts more extractive effort e_d to maximize its own payoff. Thus, $e_d > e_c$. The production yield is given by the Cobb-Douglas function of decreasing returns: $F = \gamma E^{\alpha} R^{\beta}$, where γ, α, and β are constant parameters. This gives a payoff of $\pi_i = e_i F/E - w e_i$ for each agent, with $i = c$, d, and w is the opportunity cost. Since the defectors violate the social norm, they experience social ostracism from their neighboring cooperators in the form of the Gompertz function: $O(k_c) = h \exp[\tau \exp(g k_c/k)]$, with h, g and τ being constant parameters, and k_c the number of cooperators linked to a defector. This renders an additional social cost to a defector, giving rise to a defector utility of $u_d(k_c) = \pi_d - O(k_c)(\pi_d - \pi_c)/\pi_d$. Such a social cost is absent in the case of a cooperator whose utility is given by $u_c = \pi_c$.

The social dynamics of the model involves each agent updating its strategy asynchronously. At each time step, an agent is randomly selected to compare its utility with a random neighbor. If the neighbor's utility is lower than its utility, the agent shall maintain its current strategy of being a cooperator or a defector, and there is no change in the system. However, if the neighbor's utility is higher, the agent would switch to the neighbor's strategy with a probability proportional to their normalized utility difference $P_{x\to y} = (u_{\text{neigh}} - u_{\text{agent}})/u_{\max}$, where $u_{\max}$ is the maximum achievable utility. In order to prevent the system from being trapped in a state where all agents adopt the same strategy, we include a mutation mechanism where a random agent is assigned an opposite strategy at a very low fixed rate.

We begin by considering an integrated society where $\mu = 0.5$. We assume that it is subjected to an external drive which is the resource inflow c. We start with the scenario where all agents are cooperators and $c = 0$. We increase c slowly and after each increment, allow the system to reach steady state. We call this the forward path or direction. While this progresses, we observe a gradual increase in the number of defectors, indicating a shift from full social cohesion of cooperation to states of greater disharmony. The latter arises from an increasing number of conflicting strategies in the population. At a critical resource inflow, phase change occurs and the system transits discontinuously from one extreme phase to another in both the social and resource variables. In the new phase, there is a larger proportion of defectors than cooperators. Social cooperation has drastically reduced while social disharmony persists. A further increase in c eventually leads to the state of all defectors, upon which social cooperation vanishes although the system has reached a state of harmony since there is no differing strategies. While the state of cooperation can be restored by decreasing c (the backward path or direction), the process now follows a different path. Thus, the system displays irreversibility in the form of a hysteresis loop. This behavior is illustrated in Fig. 1(a).

By reducing μ, community structure appears. As the society becomes more segregated with a smaller μ, we observe that social cooperation collapses earlier in the forward path. For weakly segregated societies ($\mu = 0.3$ and $\mu = 0.25$), the collapse of social cooperation occurs concomitantly in the two communities. When the segregation becomes sufficiently large ($\mu = 0.2$ and $\mu = 0.15$), we observe the interesting phenomenon of a double phase transition. Specifically, for the more segregated society at $\mu = 0.15$, the first phase transition happens earlier while the second phase transition occurs much later compared to $\mu = 0.2$ as c increases. A closer examination in terms of single realization [see inset of Fig. 1(a)] shows that cooperation did not collapse globally but instead at the local community level, e.g., community 2 (the red curve) collapses first. In other words, social cooperation is maintained at the community level before it is eventually destroyed at a second phase transition. It is important to



(a) (b)

FIG. 1. (a) The effect of community segregation on social cooperation driven by resource inflow ($0 \leq c \leq 60$) for a network ($N = 100$, $\langle k \rangle = 45$) with different mixing parameters: $\mu = 0.5$ (square), 0.3 (circle), 0.2 (triangle), and 0.15 (diamond). Results are plotted after averaging over 100 realizations. Inset: Single realization ($\mu = 0.2$) on P_c in community 1 (red line, left), and community 2 (blue line, right). (b) The associated resource level driven by c. Inset: Single realization ($\mu = 0.2$) of total payoff in community 1 (red line, higher value), and community 2 (blue line, lower value). Note that the solid (stable) and dotted (unstable) lines in the main figures are analytical approximations for $\mu = 0.2$.

note that the order of community collapse is purely random as it depends on the manner in which the defectors spread in each community. Like the case of a fully integrated society, the system would not reverse its path as we reduce c from the all defectors state. There is again hysteresis, but now a double phase transition back to the cooperative state even for the weakly segregated societies.

Next, we perform a detailed quantitative analysis on our coupled social-resource system. First, we solve for the fixed point of the resource dynamics and obtain

$$R^* = -E + \left(\sqrt{E^2 + 4c\frac{d}{R_{\max}}}\right)\frac{R_{\max}^2}{2d}. \qquad (1)$$

Second, we construct a master equation of the probability of P_c at time t, i.e., $P(P_c, t)$. The master equation takes the following form:

$$\frac{d}{dt}P(P_c, t) = -\frac{d}{dP_c}\{P(P_c, t)[T^+(P_c) - T^-(P_c)]\}, \quad (2)$$

where $T^+(P_c)$ ($T^-(P_c)$) is the probability that the number of cooperators increases (decreases) by one. The presence of community structure is indicated by P_{ci} (P_{di}), which denotes the probability that a cooperator (defector) is being found in community i (with $i = 1, 2$). Note that $P_{ci} + P_{di} = 1$. We define the conditional probability $q_{ci|dj}$ as the probability that a defector in community j has a neighboring cooperator in community i, where i, $j = 1, 2$. Then, by means of the Bayesian identity $P_{ci,dj} = q_{ci|dj}P_{dj} = q_{dj|ci}P_{ci}$ and the condition of equilibrium $T^+(P_c) - T^-(P_c) = 0$, we obtain the fixed points of the social dynamics by solving the following equation: $(1-\mu)\sum_{i=j}[P_{ci,dj}(u_{ci} - u_{dj})/u_{\max}] + \mu\sum_{i\neq j}[P_{ci,dj}(u_{ci} - u_{dj})/u_{\max}] = 0$, where $P_{ci,dj}$ is the probability that a cooperator in community i is connected to a defector in community j. Let us assume that a defector from community i is connected to l_{ci} neighboring cooperators from the same community and m_{cj} neighboring cooperators from a different community, i.e., $i \neq j$ here. The frequency of such a configuration is given by $B_{l_{ci}}(q_{ci|di})B_{m_{cj}}(q_{cj|di})$, which is described by the binomial distribution $B_i(q) = k!q^i(1-q)^{k-i}/[i!(k-i)!]$. By substituting this frequency and the definition of the utility, we obtain

$$\frac{\pi_d - \pi_c}{\pi_d}\sum_{i\neq j} I_{ij} = 0, \qquad (3)$$

where $I_{ij} = [(1-\mu)P_{ci,di} + \mu P_{cj,di}](\langle O(i,j)\rangle - \pi_d)$ and $\langle O(i,\ j)\rangle = \sum_{l_{ci}=0}^{(1-\mu)\langle k\rangle}\sum_{m_{cj}=0}^{\mu\langle k\rangle} B_{l_{ci}}(q_{ci|di})B_{m_{cj}}(q_{cj|di}) O(l_{ci} + m_{cj})$.

In order to solve the above equations, we have applied the pair approximation [29]. Based on detailed balance, we obtain $q_{ci|di} = [(1-\mu)\langle k\rangle - 2]P_{ci}/[(1-\mu)\langle k\rangle - 1]$ for the intracommunity link. On the other hand, the conditional probability of the intercommunity link is equal to the probability of the strategy in the external community, i.e., $q_{ci|dj} = P_{ci}$ with $i \neq j$. By inputting these conditional probabilities into Eq. (3) and using Eq. (1), we would be able to solve for P_c and c. To achieve this, we note that $P_c = (P_{c1} + P_{c2})/2$ as the two communities are identical.

The solution of these analytical approximations is illustrated in Fig. 1 as solid and dotted lines which represent the stable and unstable states, respectively. The analytical result illustrates the presence of a new phase of stable states: semi-cooperative (SC) at the societal level, sandwiched between the phases of cooperator-dominant and defector-dominant stable states. This additional phase accounts for the observation of double phase transition. If we were to trace the analytical curve by adjusting c, we would encounter phase transition between the stable phases at the fold bifurcation. A careful examination indicates that the new SC stable phase enables the system to display a double hysteresis loop. Figure 1 illustrates a comparison of our analytical approximations against the numerical results, which shows good correspondence.

Let us now analyze the features of this new SC phase in a strongly segregated society. In this phase, a strongly segregated society self-organizes into communities with different social behavior as it exploits a common pool resource. In our case of two communities, it is a community of cooperators and a community of defectors. Notably, the latter community is observed to enjoy a higher payoff by free-riding on the former community which serves to maintain the resource even though its payoff has been dampened by the defective act of the latter community. The defective act has not only significantly reduced the resource, it has also created an inequality of payoff between community 1 and 2 [Inset of Fig. 1(b)]. We examine this social aspect more closely by quantifying the social disharmony within the society. We differentiate between two types of social disharmony: internal and external. Internal (external) social disharmony is measured by the number of internal (external) cooperator-to-defector (CD) links divided by the total number of intra- (inter-) community links. It is interesting that internal social disharmony is low in this SC phase where social cooperation has collapsed in one community but maintained in the other. While the external social disharmony is high, the low internal social disharmony has enabled this society to sustain social cohesion in one community further than that of a fully integrated society as the system is being driven externally. Note that such a disparity in disharmony is absent in a fully integrated society (see Fig. 2).

Next, we apply our results to the Balinese subak. A subak is an agrarian society of farmers who exploit water, which is a common pool resource, for irrigation and rice production activities. It is a self-organized society without centralized governance, and the relation between its social and resource context fulfills the basic requirements of our model [30,31]. In an earlier pilot study, Lansing *et al.* used principal component and partial hierarchical clustering to analyze survey data from 83 farmers in 8 subaks, which experience similar social and environmental conditions, but respond to them in different ways corresponding to the cooperator-dominant and defector-dominant phases in the model [32]. Using the same methods in a follow-up survey of 20 subaks and a total of 493 farmers by Lansing, we uncovered three clusters of 19 descriptors which correspond to the three phases in our two-community coupled social-resource model. The first phase is associated with the cooperative phase since the relevant descriptors indicate strong social cooperation with low resource usage. The second phase consists of a community of defectors, where

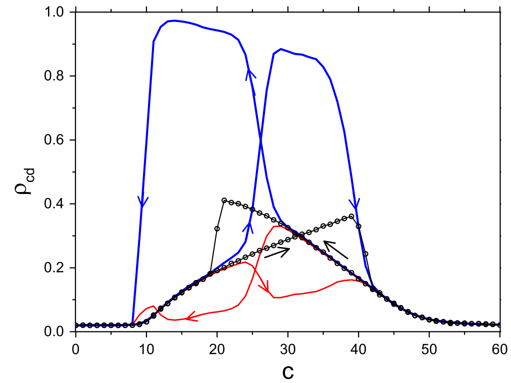


FIG. 2. The variation of internal (red line, lower value) and external (blue line, higher value) disharmony against resource inflow ($0 \le c \le 60$) with a network ($N = 100$, $\langle k \rangle = 45$) of two communities ($\mu = 0.2$). The social disharmony for an integrated society ($\mu = 0.5$) is also plotted (black circle).

the descriptors highlight low cooperation and high resource availability. The third phase is related to the SC phase with the subaks segregating into communities of contrasting social behavior as a result of caste differences. Here, we observe constant social conflicts, with moderate cooperativity and resource usage. By means of a more direct inference of the descriptors to c, P_c and ρ_{CD}, we are able to map each subaks to their respective phases as shown in Fig. 3 (see Supplemental Material [33]). In particular, our statistical analysis shows that the empirical and model

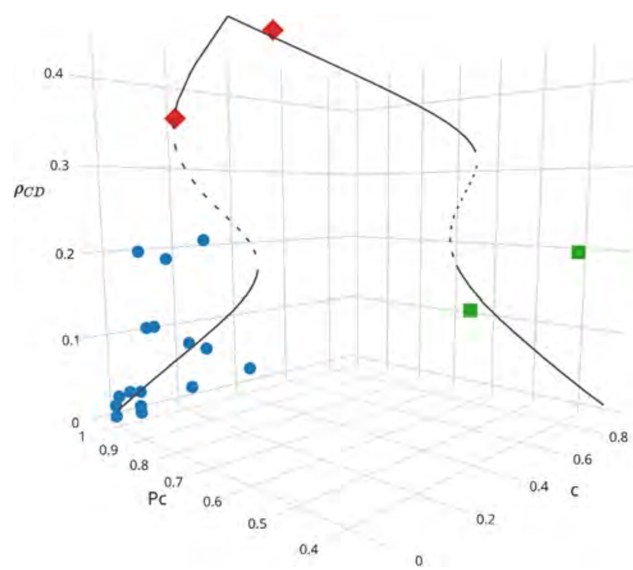


FIG. 3. Close correspondence of three clusters of subaks to the three phases indicated by the analytical curves: cooperation (circle); disharmony (diamond); and defection (square). Note that ρ_{CD} here is the fraction of external CD link as defined in the text. Statistical analysis between empirical data and the model gives the following results: $R^2 = 0.8023$; RMSD $= 0.0175$; and through the binomial test, only two subaks—Tampuagan Hilir and Selukat—are rejected based on a p value less than 0.05 (see Supplemental Material [33]).

results are close to each other and possess the same relative trend, with 90% of the subaks having a good fit to the model at a significance level of 0.05.

While the results discussed above are for a two-community society, they remain valid for a society with more than two communities; with different mean degree; when the communities possess heterogeneities and are nonidentical (refer to the Supplemental Material [33]). In these more general circumstances, we observe the occurrence of multiple step-sized phase transitions and multiple hysteresis loops. Similarly, social cooperation, resource utilization, and societal payoff, which are optimal on average when the society is integrated, are worse off under social segregation in the multiple communities. Nonetheless, the trend is not downwards all the way. Beyond a certain degree of segregation, we again observe the emergence of new stability phases. These phases possess social cohesiveness at the community level which boosts up these indicators at the societal level, albeit at a social cost of greater intercommunity conflicts. We perceive these results to be of general significance for the understanding and management of coupled social-resource systems.

J. S. Lansing's research on the Balinese subaks was supported by the cultural anthropology program of the National Science Foundation through Grant No. 3094600, and through the Complexity Institute of Nanyang Technological University Start-Up Grant No. M4081237. The conclusions and recommendations expressed here do not necessarily reflect the views of these foundations. Permission for human subjects research in Bali was granted to Lansing by the Indonesian Ministry of Research and Technology and the Human Subjects Protection Program of the University of Arizona. The Complexity Institute of Nanyang Technological University facilitated collaboration by the authors.

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[1] L. P. Kadanoff, J. Stat. Phys. **137**, 777 (2009).
[2] W. Kohn, Rev. Mod. Phys. **71**, S59 (1999).
[3] R. V. Sole, *Phase Transitions*, Primers in Complex Systems (Princeton University Press, Princeton, NJ, 2011).
[4] R. V. Solé, S. C. Manrubia, B. Luque, J. Delgado, and J. Bascompte, Complexity **1**, 13 (1996).
[5] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).
[6] B. S. Kerner, Phys. Rev. Lett. **81**, 3797 (1998).
[7] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, Rev. Mod. Phys. **87**, 925 (2015).
[8] R. Pastor-Satorras and A. Vespignani, Phys. Rev. Lett. **86**, 3200 (2001).
[9] S. Strogatz, *Sync: The Emerging Science of Spontaneous Order* (Hyperion, NY, 2003).
[10] M. Scheffer, S. Carpenter, J. A. Foley, C. Folke, and B. Walker, Nature (London) **413**, 591 (2001).
[11] M. Scheffer *et al.*, Science **338**, 344 (2012).
[12] M. Scheffer, J. Bascompte, W. A. Brock, V. Brovkin, S. R. Carpenter, V. Dakos, H. Held, E. H. Van Nes, M. Rietkerk, and G. Sugihara, Nature (London) **461**, 53 (2009).
[13] R. M. May, S. A. Levin, and G. Sugihara, Nature (London) **451**, 893 (2008).
[14] A. G. Haldane and R. M. May, Nature (London) **469**, 351 (2011).
[15] I. A. van de Leemput, M. Wichers, A. O. J. Cramer, D. Borsboom, F. Tuerlinckx, P. Kuppens, E. H. van Ness, W. Viechtbauer, E. J. Giltay, S. H. Aggen, C. Derom, N. Jacobs, K. S. Kendler, H. L. J. van der Maas, M. C. Neale, F. Peeters, E. Thiery, P. Zachar, and M. Scheffer, Proc. Natl. Acad. Sci. U.S.A. **111**, 87 (2014).
[16] H. S. Sugiarto, N. N. Chung, C. H. Lai, and L. Y. Chew, Phys. Rev. E **91**, 062804 (2015).
[17] N. N. Chung, L. Y. Chew, and C. H. Lai, Europhys. Lett. **104**, 28003 (2013).
[18] S. J. Lade, A. Tavoni, S. A. Levin, and M. Schlüter, Theor. Ecol. **6**, 359 (2013).
[19] E. Ostrom, *Governing the Commons: The Evolution of Institutions for Collective Action (Political Economy of Institutions and Decisions)* (Cambridge University Press, Cambridge, England, 1990).
[20] W. Steffen, P. J. Crutzen, and J. R. McNeill, AMBIO: A Journal of the Human Environment **36**, 614 (2007).
[21] G. Ceballos, P. R. Ehrlich, A. D. Barnosky, A. García, R. M. Pringle, and T. M. Palmer, Sci. Adv. **1**, e1400253 (2015).
[22] P. M. Vitousek, H. A. Mooney, J. Lubchenco, and J. M. Melillo, Science **277**, 494 (1997).
[23] T. C. Schelling, Am. Econ. Rev. **59**, 488 (1969).
[24] T. C. Schelling, J. Math. Sociol. **1**, 143 (1971).
[25] M. McPherson, L. Smith-Lovin, and J. M. Cook, Annu. Rev. Sociol. **27**, 415 (2001).
[26] M. Girvan and M. E. J. Newman, Proc. Natl. Acad. Sci. U.S.A. **99**, 7821 (2002).
[27] A. Lancichinetti, S. Fortunato, and F. Radicchi, Phys. Rev. E **78**, 046110 (2008).
[28] A. Tavoni, M. Schlüter, and S. Levin, J. Theor. Biol. **299**, 152 (2012).
[29] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature (London) **441**, 502 (2006).
[30] J. S. Lansing and K. M. Fox, Phil. Trans. R. Soc. B **366**, 927 (2011).
[31] J. S. Lansing, *Perfect Order: Recognizing Complexity in Bali* (Princeton University Press, Princeton, NJ, 2006).
[32] J. S. Lansing, S. A. Cheong, L. Y. Chew, M. P. Cox, M.-H. Ringo Ho, and W. A. Arthawiguna, Current anthropology **55**, 232 (2014).
[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.118.208301 for additional analysis on more than two communities that are nonidentical, and for the description of the empirical data which includes a statistical analysis between data and model.